

[Time: 3:00 Hrs.]

[Marks: 80]

please check Whether you have got the right question paper.

N.B: 1.All questions are compulsory.

2. Figures to right indicate full marks.

3. Scientific calculator can be used.

Q.1 a) Let $T: v \rightarrow v$ be a linear transformation then the following are equivalent **10**

(i) T is Orthogonal linear transformation

(ii) $\|T(x)\| = \|x\|, \forall x \in v$

b) Attempt **any Two** of the following: **10**

i) Define Isometry in \mathbb{R}^n and show that rotation in \mathbb{R}^3 is an isometry. **5**

ii) Find the equation of plane contains the point $P(2,0,0)$, $Q(0,2,1)$ & $R(1,0,3)$ **5**

iii) Find a vector tangent to the line of intersection of the planes $2x+y-3z=2$, $-x+2y-z=1$ and find the angle between them. **5**

Q.2 a) A parametric curve has a unit speed Reparametrization if and only if it is Regular **10**

b) Attempt **any Two** of the following: **10**

i) A parametric curve $r(t)$ has the property that $\frac{d^2\gamma(t)}{dt^2} = 0, \forall t$, what can we say about parametric curve? **5**

ii) Show that any reparametrization of regular curve is regular. **5**

iii) Define the arc length of regular parametric curve $\gamma(t)$ and also find the arc of logarithmic spiral $(e^t \cos t, e^t \sin t)$ starting at $t=0$ **5**

Q.3 a) Write a parametric equation of sphere **10**

$S^2 = \{(x, y, z) \in \mathbb{R}^2 : x^2 + y^2 + z^2 = 1\}$ and also show that sphere is a regular surface.

- b) Attempt **any Two** of the following: 10
- i) Define Orienable surface and find the equation of tangent to the surface patches $\sigma(\gamma, \theta) = (\gamma \cosh \theta, \gamma \sinh \theta, r^2)$ at $(1, 0, 1)$. 5
- ii) Show that plane in \mathbb{R}^3 is a regular surface. 5
- iii) Consider a smooth surface S defined by $f(x, y, z) = 0$, where f is a smooth function such that $\nabla f = (f_x, f_y, f_z)$ is perpendicular to the tangent plane at every point of S and hence show that surface S is Orienable. 5

- Q.4** a) Find the first fundamental form of surface patch σ and find first fundamental form of sphere $\sigma(\theta, \phi) = (\cos \theta \cos \phi, \cos \theta \sin \phi, \sin \theta)$ 10
- b) Attempt **any Two** of the following: 10
- i) Define the area of surface patch σ and if E, F and G are coefficient of first fundamental form of surface patch σ then $\|\sigma_u X \sigma_v\| = (EG - F^2)^{\frac{1}{2}}$ 5
- ii) State and prove Euler's formula for the normal curvature 5
- iii) Prove that all curves lying on surface S and having a point $p \in S$ at the same tangent line have the same normal curvature? 5
